orbit

Given a space X let $\mathcal{H}(X)$ stand for the group of autohomeomorphisms of X. If a point $p \in X$ is fixed, then $\{h(p) \in X : h \in \mathcal{H}(X)\}$ is called an *orbit of* p. Orbits of points of X either are mutually disjoint or coincide, and their union is the whole X.

order

Let (X, T) be a topological space, and let $A \subset X$. Let β be a cardinal number. We say that A *is of order less than equal to* β in X, written $\operatorname{ord}(A, X) \leq \beta$, provided that for each $U \in T$ such that $A \subset U$, there exists $V \in T$ such that $A \subset V \subset U$ and $|Bd(V)| \leq \beta$.

We say that A *is of order* β *in* X, written $\operatorname{ord}(A, X) = \beta$, provided that $\operatorname{ord}(A, X) \leq \beta$ and $\operatorname{ord}(A, X) \not\leq \alpha$ for any cardinal number $\alpha < \beta$.

A concept of an *order* of a point p in a continuum X (in the sense of Menger-Urysohn), written $\operatorname{ord}(p, X)$, is defined as follows. Let $\mathfrak{n}$ stand for a cardinal number. We write:

$\operatorname{ord}(p, X) \leq \mathfrak{n}$ provided that for every $\varepsilon > 0$ there is an open neighborhood U of p such that $diam \leq \varepsilon$ and $\operatorname{card} \operatorname{bd} U \leq \mathfrak{n}$;

$\operatorname{ord}(p, X) = \mathfrak{n}$ provided that $\operatorname{ord}(p, X) \leq \mathfrak{n}$ and for each cardinal number $\mathfrak{m} < \mathfrak{n}$ the condition $\operatorname{ord}(p, X) \leq \mathfrak{m}$ does not hold;

$\operatorname{ord}(p, X) = \omega$ provided that the point p has arbitrarily small open neighborhoods U with finite boundaries $\operatorname{bd} U$ and $\operatorname{card} \operatorname{bd} U$ is not bounded by any $n \in \mathbb{N}$.

Thus, for any continuum X we have

$$\operatorname{ord}(p, X) \in \{1, 2, \ldots, n, \ldots, \omega, \aleph_0, 2^{\aleph_0}\}$$

(convention: $\omega < \aleph_0$); see [55, §51, I, p. 274].

Let a dendroid X and a point $p \in X$ be given. Then p is said to be a *point of order at least* $\mathfrak{m}$ *in the classical sense* provided that p is the center of an $\mathfrak{m}$-od contained in X. We say that p is a *point of order* $\mathfrak{m}$ *in the classical sense* provided that $\mathfrak{m}$ is the minimum cardinality for which the above condition is satisfied (see [8, p. 229]).

order preserving mapping

For each point p of a continuum X equipped with an arc-structure $A : X \times X \to C(X)$ we define a partial order $\leq_p$ by letting $x \leq_p y$ whenever $A(p, x) \subset A(p, y)$. Let X and Y be continua with fixed arc-structures A and B, respectively. We say that a surjective mapping $f : X \to Y$ is a $\leq_p$-*mapping* provided that $x \leq_p y$ in X implies that $f(x) \leq_{f(p)} f(y)$ in Y. If, in addition, $Y \subset X$, $B = A|(Y \times Y)$, and f is a retraction, then f is called a $\leq_p$-*retraction* (or $\leq_p$-*preserving retraction*). The concept of a $<_p$-*mapping* is defined in a similar manner (with $f(x) \neq f(y)$ implied by $x \neq y$). For *order preserving mappings* see e.g. [42, I.7, p. 553].