Chapter 14

New examples

New section contains new examples.

14.1 New examples

New examples are nice.

14.1.1 Whyburn's Curve

The well-known Jordan Curve Theorem says that a simple closed curve in the plane cuts the plane into two regions and is their common boundary. Thus, it is a cutting of the plane. The so called Warsaw Circle, obtained from the $\sin(1/x)$-curve

$$S = \{(0, y) \in \mathbb{R}^2 : y \in [-1, 1]\} \cup \{(x, \sin(1/x)) \in \mathbb{R}^2 : x \in (0, 1]\}$$

by identifying the points $(0, -1)$ and $(1, \sin 1)$ also is a cutting. But both these continua contain arcs, which certainly do not cut the plane. So, one can ask a question if there exists such a cutting $X \subset \mathbb{R}^2$ that each nondegenerate subcontinuum of X also cuts $\mathbb{R}^2$. The question has been answered in the affirmative in 1930 by G. T. Whyburn who constructed in [3] a *hereditary cutting of the plane*, which is now called the *Whyburn's Curve*. The present description of this peculiar continuum is a modification of the original one due to A. Lelek [1], to obtain some extra properties of the curve, and it follows Lelek's sketch of the construction in [3, p. 117-119]. We say that a continuum X lying in the plane $\mathbb{R}^2$ cuts the plane between points a and b if the points belong to different components of $\mathbb{R}^2 \setminus X$. A continuum X is a cutting of $\mathbb{R}^2$ if there are some two points such that X cuts $\mathbb{R}^2$ between them. Take the $\sin(1/x)$curve S and identify points $(0, 1)$ and $(0, -1)$ into one point r. The obtained curve (still considered as a subset of the plane) consists of a circle C and a sinusoidal curve approximating the circle, so it is a cutting of the plane. If we add to the curve an arc starting at r and having only this point in common with the curve, we